Section 11.1

## Polar Coordinates and Graphs

Objectives:

1. Graph polar functions and equations.
2. Use tests to determine if polar functions and equations exhibit symmetry.

WHO IS THIS MAN?


## Polar Coordinates



We know how to locate points on the plane by using a rectangular coordinate system.

Graph ( $-4,4 \sqrt{ } 3$ ).
How many different ways can we name this point?

## Polar Coordinates



There is another way we can locate points on the plane, and that is by using the polar coordinate system.

O is a fixed point called the pole. The polar axis is a horizontal ray emanating to the right from the pole.

Show
rectangular grid

Show polar system

## Polar Coordinates



Show polar

grid

## Polar Coordinates



## Polar Coordinates



## Polar Coordinates



Show Rectangular Grid
Hide Rectangular Grid

Show Polar Grid
Hide Polar Grid

How could we find the rectangular coordinates of the point $\left(4,60^{\circ}\right)$ ?

## Polar Coordinates



Find the rectangular coordinates of the point (3, $-135^{\circ}$ ).

## Polar Coordinates

In general, if we have the polar coordinates of a point $(r, \theta)$, how do we find the rectangular coordinates ( $\mathrm{x}, \mathrm{y}$ )?

$$
x=
$$

$$
y=
$$



## Polar Coordinates

Now suppose we have the rectangular coordinates of a point ( $x, y$ ), and we wish to change to the polar coordinates $(r, \theta)$.

$$
r=\quad \theta=
$$



## Polar Coordinates



Find the polar coordinates of the point (6, -2 ).

[^0]Hide Polar Grid

## Polar Coordinates

Points aren't very interesting by themselves. Descartes invented his coordinate so that he would have an algebraic method to describe curves.

We can also describe curves in the polar coordinate system.

Write a polar equation for the line $x-2 y=4$.
Solve your equation for r.

## Polar Coordinates

This is way more complicated than the rectangular equation was! We will, however, see curves where the polar representation is much simpler than the rectangular representation.

## Polar Graphs

Sketch the graphs:
$y=0$

$r=0$


## Polar Graphs

Sketch the graphs:

$$
y=3
$$

$$
r=3
$$




## Polar Graphs

Make a conjecture about the graphs of $y=$ constant and $r=$ constant, and the relationship between the graphs. Use the Sketchpad file to help you with your conjectures.

## Polar Graphs

Sketch the graphs:

$$
y=x
$$

$$
r=\theta
$$




## Polar Graphs

Sketch the graphs:

$$
r=-\theta
$$



## Polar Graphs

Make a conjecture about the graphs of $y=a \cdot x$ and $r=a \cdot \theta$, and the relationship between the graphs. Use the Sketchpad file to help you with your conjectures.

## Polar Graphs

Let's explore the graphs of $r=a \cdot \cos \theta$ and $r=a \cdot \sin \theta$.

## Polar Graphs

These graphs certainly look like circles. Convert the equations $r=a \cdot \cos \theta$ and $r=a \cdot \sin \theta$ into rectangular coordinates.

## Polar Graphs

What would these equations look like in standard form?

## Polar Graphs

Let's look at the graphs of these equations.

## Polar Graphs

Do you see any symmetry in the graph of $r=\operatorname{acos} \theta$ ?


## Polar Graphs



It's easy to see why, since $\cos \theta=\cos (-\theta)$.

## Symmetry Test \#1

## Polar Graphs

Do you see any symmetry in the graph of $r=\operatorname{asin} \theta$ ?


## Polar Graphs



There are two tests for $\theta=\pi / 2$ symmetry:

Symmetry Test \#2

## Polar Graphs



There are two tests for $\theta=\pi / 2$ symmetry:

## Symmetry Test \#3

## Polar Graphs

The other type of symmetry we will be concerned with is polar (origin) symmetry. We'll see examples of graphs that exhibit this symmetry a little later. For now, the test is:


Symmetry Test \#4

## Polar Graphs



## Polar Graphs

It is also possible to define a function in polar coordinates.

Recall our formal definition for a function in rectangular coordinates:

Define a function in polar coordinates:

## Polar Graphs

Based on that definition, are $r=\operatorname{acos} \theta$ and $r=\operatorname{asin} \theta$ functions on the polar plane?

Are the rectangular equations that give the same graphs on the rectangular plane functions?

## Polar Graphs

Now let's look at this family of equations:

$$
r=a+b \sin \theta
$$

Are these equations functions?

What symmetries do these graphs have?

## Polar Graphs

Sketch the graphs:
$y=3+2 \sin x$



## Polar Graphs

Sketch the graphs:

$$
y=2+2 \sin x
$$




## Polar Graphs

Sketch the graphs:
$y=1+2 \sin x$


$$
r=1+2 \sin \theta
$$



## Polar Graphs

So, for the graphs of $r=a+b \sin \theta$, what happens if:
$\mathrm{a}>\mathrm{b}$
$a=b$

$$
a<b
$$

## Polar Graphs

Let's verify our conjectures about the graphs of $y=a+b \sin x$ and $r=a+b \sin \theta$.

## Polar Graphs

Sketch the graph of $r=-2+3 \sin \theta$.



## Polar Graphs

Compare and contrast the graphs of the following equations:

$$
\begin{aligned}
& r=2+3 \sin \theta \quad r=-2+3 \sin \theta \\
& r=2-3 \sin \theta \quad r=-2-3 \sin \theta
\end{aligned}
$$

## Polar Graphs

Sketch the graphs:

$$
y=3 \cos (2 x)
$$

$$
r=3 \cos (2 \theta)
$$




## Polar Graphs

Sketch the graphs:

$$
y=3 \cos (3 x)
$$


$r=3 \cos (3 \theta)$


## Polar Graphs

Sketch the graphs:

$$
y=3 \cos (4 x)
$$


$r=3 \cos (4 \theta)$


## Polar Graphs

In the graph of $r=a \cos (b \theta)$ :

What is the role of $a$ ?

What is the role of $b$ ?

## Polar Graphs

Let's verify our conjectures about the graphs of $y=a \cos (b x)$ and $r=a \cos (b \theta)$.


## Polar Graphs

Sketch the graphs:
$y^{2}=4 \sin (2 x)$



## Polar Graphs

Sketch the graphs:
$y^{2}=9 \cos (3 x)$

$r^{2}=9 \cos (3 \theta)$


## Polar Graphs

In the graph of $r^{2}=a \cos (b \theta)$ :

What is the role of $a$ ?

What is the role of $b$ ?

## Polar Graphs

Let's verify our conjectures about these graphs.



[^0]:    Hide Rectangular Grid

